Snell’s Law for Nonlinear Beams

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The refraction of plane waves at a uniform boundary between dissimilar linear dielectric materials is perhaps one of the oldest and best well-understood phenomenon in optics. In contrast, the behaviour of a beam with a finite transverse cross-section is much less well understood. The situation is even more complicated when the two materials in question are nonlinear in nature. Seminal analyses some two decades ago considered scalar spatial optical solitons incident on the boundary between two dissimilar Kerr-type media [1]. While these analyses were highly instructive, they were based upon a governing nonlinear Schrödinger equation whose central tenet was that angles of incidence, reflection and refraction must be negligibly small. In practice, this is not entirely satisfactory since one would like these angles to be unconstrained.

To this end, we have recently developed a theory describing the refraction of soliton beams incident at any angle on a Kerr-type interface [2]. Our latest efforts have been to extend our Kerr analyses to a much wider class of power-law materials, of which the Kerr nonlinearity is a particular case. By a curious twist of fate, it turns out that the full nonlinear-beams problem can be described by a simple generalization of the trivially-familiar Snell’s law. Figure 1 shows excellent agreement between theoretical predictions and full simulations of the interface problem.

![Figure 1](image_url)

Figure 1. (Left) Typical interface geometry and (right) predictions of soliton refraction using the new Snell law, \( \gamma n_0 \cos \theta = n_0 \cos \theta_i \), where \( \gamma \) is a universal function of system and beam parameters.

References