Basic Mathematics

Algebraic Fractions

R Horan & M Lavelle

The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence in the use of algebraic fractions.

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Table of Contents

1. Algebraic Fractions (Introduction)
2. Addition of Algebraic Fractions
3. Simple Partial Fractions
4. Quiz on Algebraic Fractions
Solutions to Exercises
Solutions to Quizzes

1. Algebraic Fractions(Introduction)

Algebraic fractions have properties which are the same as those for numerical fractions, the only difference being that the the numerator (top) and denominator (bottom) are both algebraic expressions.

Example 1 Simplify each of the following fractions.

$$\textbf{(a)}\quad \frac{2b}{7b^2}\,, \qquad\qquad \textbf{(b)}\quad \frac{3x+x^2}{6x^2}\,.$$

Solution

(a)

$$\frac{2b}{7b^2} = \frac{2\times \cancel{b}}{7\times b\times \cancel{b}} = \frac{2}{7b}$$

(b)

$$\begin{aligned}\frac{3x+x^2}{6x^2} &= \frac{x\times(3+x)}{x\times 6x}\\ &= \frac{\cancel{x}\times(3+x)}{\cancel{x}\times 6x} = \frac{3+x}{6x}\end{aligned}$$

N.B. The cancellation in **(b)** is allowed since x is a common factor of the numerator and the denominator.

Sometimes a little more work is necessary before an algebraic fraction can be reduced to a simpler form.

Example 2 Simplify the algebraic fraction

$$\frac{x^2-2x+1}{x^2+2x-3}$$

Solution

In this case the numerator and denominator can be factored into two terms, thus

$$x^2-2x+1=(x-1)^2\,, \quad \text{and} \quad x^2+2x-3=(x-1)(x+3)\,.$$

(See the package on **factorising expressions**). With this established the simplification proceeds as follows:

$$\begin{aligned}\frac{x^2-2x+1}{x^2+2x-3} &= \frac{(x-1)\times(x-1)}{(x+3)\times(x-1)}\\ &= \frac{x-1}{x+3} \quad \text{(cancelling } (x-1))\end{aligned}$$

EXERCISE 1. Simplify each of the following algebraic fractions. (Click on the green letters for solution.)

(a) $\dfrac{8y}{2y^3}$ (b) $\dfrac{2y}{4x}$ (c) $\dfrac{7a^6b^3}{14a^5b^4}$

(d) $\dfrac{(2x)^2}{4x}$ (e) $\dfrac{5y+2y^2}{7y}$ (f) $\dfrac{5ax}{15a+10a^2}$

(g) $\dfrac{2z^2-4z}{2z^2-10z}$ (h) $\dfrac{y^2+7y+10}{y^2-25}$ (i) $\dfrac{w^2-5w-14}{w^2-4w-21}$

Now try this short quiz.

Quiz Which of the following is a simplified version of

$$\frac{t^2+3t-4}{t^2-3t+2}\ ?$$

(a) $\dfrac{t-4}{t-2}$ (b) $\dfrac{t-4}{t+2}$ (c) $\dfrac{t+4}{t-2}$ (d) $\dfrac{t+4}{t+2}$

So far, simplification has been achieved by cancelling common factors from the numerator and denominator. There are fractions which can be simplified by *multiplying* the numerator and denominator by an appropriate common factor, thus obtaining an equivalent, simpler expression.

Example 3 Simplify the following fractions.

(a) $\dfrac{\frac{1}{4}+y}{\frac{1}{2}}$ **(b)** $\dfrac{3x+\frac{1}{x}}{2}$

Solution

(a) In this case, multiplying both the numerator and the denominator by 4 gives:

$$\frac{\frac{1}{4}+y}{\frac{1}{2}} = \frac{4\left(\frac{1}{4}+y\right)}{4\left(\frac{1}{2}\right)} = \frac{1+4y}{2}$$

(b) To simplify this expression, multiply the numerator and denominator by x. Thus

$$\frac{3x+\frac{1}{x}}{2} = \frac{x\left(3x+\frac{1}{x}\right)}{2x} = \frac{3x^2+1}{2x}$$

Now try this exercise on similar examples.

EXERCISE 2. Simplify each of the following algebraic fractions. (Click on the green letters for solution.)

(a) $\dfrac{4y-\frac{3}{2}}{2}$ (b) $\dfrac{2x+\frac{1}{2}}{x+\frac{1}{4}}$ (c) $\dfrac{z-\frac{1}{3}}{z-\frac{1}{2}}$

(d) $\dfrac{2-\frac{1}{x}}{2}$ (e) $\dfrac{3t-\frac{2}{t}}{\frac{1}{2}}$ (f) $\dfrac{z-\frac{1}{2z}}{z-\frac{1}{3z}}$

For the last part of this section, try the following short quiz.

Quiz Which of the following is a simplified version of

$$\frac{x-\frac{1}{x+1}}{x-1}\ ?$$

(a) $\dfrac{x^2-x+1}{x^2+x+1}$ (b) $\dfrac{x^2-x+1}{x^2-1}$ (c) $\dfrac{x^2-1}{x^2-x-1}$ (d) $\dfrac{x^2+x-1}{x^2-1}$

2. Addition of Algebraic Fractions

Addition (and subtraction) of algebraic fractions proceeds in exactly the same manner as for numerical fractions.

Example 4 Write the following sum as a single fraction in its simplest form.

$$\frac{2}{x+1}+\frac{1}{x+2}$$

Solution The *least common multiple* of the denominators (see the package on **fractions**) is $(x+1)(x+2)$. Thus

$$\begin{aligned}\frac{2}{x+1}+\frac{1}{x+2} &= \frac{2\times(x+2)}{(x+1)\times(x+2)}+\frac{1\times(x+1)}{(x+2)\times(x+1)}\\ &= \frac{2x+4}{(x+1)(x+2)}+\frac{x+1}{(x+1)(x+2)}\\ &= \frac{(2x+4)+(x+1)}{(x+1)(x+2)}=\frac{3x+5}{(x+1)(x+2)}\end{aligned}$$

EXERCISE 3. Evaluate each of the following fractions. (Click on the green letters for solution.)

(a) $\dfrac{2}{y} + \dfrac{3}{z}$

(b) $\dfrac{1}{3y} - \dfrac{2}{5y}$

(c) $\dfrac{3z+1}{3} - \dfrac{2z+1}{2}$

(d) $\dfrac{3t+1}{2} + \dfrac{1}{t}$

(e) $\dfrac{x+1}{2} + \dfrac{1}{x-1}$

(f) $\dfrac{2}{w+3} - \dfrac{5}{w-1}$

Quiz Which of the following values of **a** is needed if

$$\frac{\mathbf{a}}{2x+1} + \frac{1}{x+2} = \frac{4x+5}{(2x+1)(x+2)}\,?$$

(a) $\mathbf{a} = 3$ (b) $\mathbf{a} = -3$ (c) $\mathbf{a} = 2$ (d) $\mathbf{a} = -2$

3. Simple Partial Fractions

The last quiz was an example of *partial fractions*, i.e. the technique of decomposing a fraction as a sum of simpler fractions. This section will consider the simpler forms of this technique.

Example 5 Find the partial fraction decomposition of $4/(x^2 - 4)$.

Solution The denominator factorises as $x^2 - 4 = (x-2)(x+2)$. (See the package on **quadratics**.) The partial fractions will, therefore, be of the form $a/(x-2)$ and $b/(x+2)$. Thus

$$\frac{a}{x-2} + \frac{b}{(x+2)} = \frac{4}{(x-2)(x+2)}$$

$$\frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x+2)(x-2)} = \frac{4}{(x-2)(x+2)}$$

$$\frac{(a+b)x + 2(a-b)}{(x-2)(x+2)} = \frac{4}{(x-2)(x+2)}$$

$$\text{so that } (a+b)x + 2(a-b) = 4$$

The last line is

$$(a + b)x + 2(a - b) = 4\,,$$

and this enables a and b to be found. For the equation to be true *for all* values of x the coefficients must match, i.e.

$$\begin{aligned} a + b &= 0 \qquad \text{(coefficients of } x) \\ 2a - 2b &= 4 \qquad \text{(constant terms)} \end{aligned}$$

where the first equation holds since there is no x term in $4/(x^2 - 4)$. This set of simultaneous equations may be solved to give $a = 1$ and $b = -1$. (See the package on **simultaneous equations** for a method of finding these solutions.)
Thus

$$\frac{4}{(x-2)(x+2)} = \frac{1}{x-2} - \frac{1}{(x+2)}$$

EXERCISE 4. For each of the following, find a and b. (Click on the green letters for solution.)

(a) $\dfrac{a}{x-2} + \dfrac{b}{x+2} = \dfrac{4x}{(x-2)(x+2)}$

(b) $\dfrac{a}{z-3} + \dfrac{b}{z+2} = \dfrac{z+7}{(z-3)(z+2)}$

(c) $\dfrac{a}{w-4} + \dfrac{b}{w+1} = \dfrac{3w-2}{(w-4)(w+1)}$

Now try this final, short quiz.

Quiz If $\dfrac{a}{2x-3} + \dfrac{b}{3x+4} = \dfrac{x+7}{(2x-3)(3x+4)}$, which of the following is the solution to the equation?

(a) $a = 1,\ b = -1$ (b) $a = -1,\ b = 1$
(c) $a = 1,\ b = 1$ (d) $a = -1,\ b = -1$

4. Quiz on Algebraic Fractions

Begin Quiz In each of the following, choose:

1. the simplified form of $(z^2 + 4z - 5)/(z^2 - 4z + 3)$

(a) $(z + 5)/(z + 3)$ (b) $(z + 5)/(z - 3)$
(c) $(z - 5)/(z + 3)$ (d) $(z - 5)/(z - 3)$

2. the sum $[1/(w - 2)] - [1/(w + 7)]$

(a) $5/(w^2 + 5w - 14)$ (b) $5/(w^2 - 5w + 14)$
(c) $9/(w^2 - 5w + 14)$ (d) $9/(w^2 + 5w - 14)$

3. a and b if
$[a/(3x + 2)] + [b/(4x - 3)] = (x - 5)/[(3x + 2)(4x - 3)]$

(a) $a = 1,\ b = 1$ (b) $a = -1,\ b = 1$
(c) $a = -1,\ b = -1$ (d) $a = 1,\ b = -1$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

$$\begin{aligned}\frac{8y}{2y^3} &= \frac{4\times 2y}{y^2\times 2y}\\ &= \frac{4\times \not{2}\times \not{y}}{y^2\times \not{2}\times \not{y}}\\ &= \frac{4}{y^2}\end{aligned}$$

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Exercise 1(b)

$$\begin{aligned}\frac{2y}{4x} &= \frac{2 \times y}{2 \times 2x} \\ &= \frac{\not{2} \times y}{\not{2} \times 2x} \\ &= \frac{y}{2x}\end{aligned}$$

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Exercise 1(c) The fraction is $\frac{7a^6b^3}{14a^5b^4}$. This time, instead of expanding the factors, it is easier to use the rule for powers

$$\frac{a^m}{a^n} = a^{m-n}\,.$$

(See the package on **powers**.) Thus

$$\begin{aligned}\frac{7a^6b^3}{14a^5b^4} &= \frac{7}{14}\times\frac{a^6}{a^5}\times\frac{b^3}{b^4}\\ &= \frac{1}{2}\times a^{6-5}\times b^{3-4}\\ &= \frac{1}{2}\times a^1\times b^{-1} = \frac{a}{2b}\end{aligned}$$

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Exercise 1(d)

$$\begin{aligned}\frac{(2x)^2}{4x} &= \frac{2\times x\times 2\times x}{2\times 2\times x}\\ &= \frac{\not{2}\times \not{x}\times \not{2}\times x}{\not{2}\times \not{2}\times \not{x}}\\ &= x\,.\end{aligned}$$

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Exercise 1(e)

$$\begin{aligned} \frac{5y+2y^2}{7y} &= \frac{y\times(5+2y)}{7\times y} \\ &= \frac{\not{y}\times(5+2y)}{7\times\not{y}} \\ &= \frac{5+2y}{7}\,. \end{aligned}$$

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Exercise 1(f)

$$\begin{aligned}\frac{5ax}{15a+10a^2} &= \frac{5\times a\times x}{5\times a\times(3+2a)}\\ &= \frac{\not 5\times \not a\times x}{\not 5\times \not a\times(3+2a)}\\ &= \frac{x}{3+2a}\end{aligned}$$

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Exercise 1(g)

$$\begin{aligned}\frac{2z^2-4z}{2z^2-10z} &= \frac{2\times z\times(z-2)}{2\times z\times(z-5)}\\ &= \frac{\cancel{2}\times \cancel{z}\times(z-2)}{\cancel{2}\times \cancel{z}\times(z-5)}\\ &= \frac{z-2}{z-5}\,.\end{aligned}$$

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Exercise 1(h) In this case, some initial factorisation is needed (see the package on **factorising expressions**).

$$y^2 + 7y + 10 = (y+5)(y+2) \quad \text{and} \quad y^2 - 25 = (y+5)(y-5)$$

Thus

$$\begin{aligned} \frac{y^2 + 7y + 10}{y^2 - 25} &= \frac{(y+5)(y+2)}{(y+5)(y-5)} \\ &= \frac{y+2}{y-5} \end{aligned}$$

where the factor $(y+5)$ has been cancelled.

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Exercise 1(i) Again, in this case, some initial factorisation is needed (see the package on **factorising expressions**).

$w^2 - 5w - 14 = (w-7)(w+2)$ and $w^2 - 4w - 21 = (w-7)(w+3)$

Thus

$$\begin{aligned} \frac{w^2 - 5w - 14}{w^2 - 4w - 21} &= \frac{(w-7)(w+2)}{(w-7)(w+3)} \\ &= \frac{w+2}{w+3}, \end{aligned}$$

where the factor $(w-7)$ has been cancelled.

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Exercise 2(a) The fraction is simplified by multiplying both the numerator and the denominator by 2.

$$\frac{4y - \frac{3}{2}}{2} = \frac{2\left(4y - \frac{3}{2}\right)}{2 \times 2} = \frac{8y - 3}{4}$$

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Exercise 2(b) This fraction is simplified by multiplying both the numerator and the denominator by 4. Thus

$$\frac{2x+\frac{1}{2}}{x+\frac{1}{4}} = \frac{4\left(2x+\frac{1}{2}\right)}{4\left(x+\frac{1}{4}\right)} = \frac{8x+2}{4x+1}$$

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Exercise 2(c) In this case, since the numerator contains the fraction $1/3$ and the denominator contains the fraction $1/2$, the common factor needed is $2 \times 3 = 6$. Thus

$$\frac{z - \frac{1}{3}}{z - \frac{1}{2}} = \frac{6\left(z - \frac{1}{3}\right)}{6\left(z - \frac{1}{2}\right)} = \frac{6z - 2}{6z - 3}$$

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Exercise 2(d) For this fraction the required multiplier is x.

$$\frac{2-\frac{1}{x}}{2}=\frac{x\left(2-\frac{1}{x}\right)}{2x}=\frac{2x-1}{2x}$$

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Exercise 2(e) Here the numerator includes the fraction $2/t$ and the denominator is the fraction $1/2$, so the required multiplier is $2t$.

$$\frac{3t - \frac{2}{t}}{\frac{1}{2}} = \frac{2t\left(3t - \frac{2}{t}\right)}{2t\left(\frac{1}{2}\right)} = \frac{6t^2 - 4}{t}$$

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Exercise 2(f) For the last part of this exercise, since the numerator includes the fraction $1/2z$ and the denominator includes the fraction $1/3z$, the common multiplier is $6z$.

$$\frac{z - \frac{1}{2z}}{z - \frac{1}{3z}} = \frac{6z\left(z - \frac{1}{2z}\right)}{6z\left(z - \frac{1}{3z}\right)} = \frac{6z^2 - 3}{6z^2 - 2}$$

Click on the green square to return □

Exercise 3(a) The *least common multiple* of the denominators is yz. Thus

$$\begin{aligned}\frac{2}{y} + \frac{3}{z} &= \frac{2 \times z}{y \times z} + \frac{3 \times y}{z \times y} \\ &= \frac{2z}{yz} + \frac{3y}{yz} \\ &= \frac{3y + 2z}{yz}\end{aligned}$$

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Exercise 3(b) Here the *least common multiple* of the denominators is $15y$, so

$$\begin{aligned}
\frac{1}{3y}-\frac{2}{5y} &= \frac{5\times 1}{5\times 3y}-\frac{3\times 2}{3\times 5y}\\
&= \frac{5}{15y}-\frac{6}{15y}\\
&= \frac{5-6}{15y} = -\frac{1}{15y}
\end{aligned}$$

Click on the green square to return □

Exercise 3(c) The *least common multiple* of the denominators of the two fractions in this case is 6. Thus

$$\begin{aligned}\frac{3z+1}{3}-\frac{2z+1}{2} &= \frac{2\times(3z+1)}{2\times 3}-\frac{3\times(2z+1)}{3\times 2}\\ &= \frac{6z+2}{6}-\frac{6z+3}{6}\\ &= \frac{(6z+2)-(6z+3)}{6}=-\frac{1}{6}\end{aligned}$$

Simplification in this case has shown that the difference of these two fractions is independent of z.

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Exercise 3(d) The *least common multiple* of the denominators of the two fractions in this case is $2t$. The sum simplifies as follows.

$$\begin{aligned}\frac{3t+1}{2}+\frac{1}{t} &= \frac{t\times(3t+1)}{t\times 2}+\frac{2\times 1}{2\times t}\\ &= \frac{3t^2+t}{2t}+\frac{2}{2t} = \frac{3t^2+t+2}{2t}\end{aligned}$$

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□

Exercise 3(e)
Here the required *least common multiple* of the denominators is the factor $2(x-1)$. Proceeding as before:

$$\begin{aligned}
\frac{x+1}{2}+\frac{1}{x-1} &= \frac{(x-1)\times(x+1)}{(x-1)\times 2}+\frac{2\times 1}{2\times(x-1)} \\
&= \frac{(x^2-1)}{2(x-1)}+\frac{2}{2(x-1)} \\
&= \frac{(x^2-1)+2}{2(x-1)} = \frac{x^2+1}{2(x-1)}
\end{aligned}$$

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Exercise 3(f)
Here the required *least common multiple* of the denominators is

$$(w+3)(w-1) = w^2 + 2w - 3\,.$$

With this in mind,

$$\begin{aligned}
\frac{2}{w+3} - \frac{5}{w-1} &= \frac{(w-1)\times 2}{(w-1)\times(w+3)} - \frac{(w+3)\times 5}{(w+3)\times(w-1)} \\
&= \frac{2w-2}{(w+3)(w-1)} - \frac{5w+15}{(w+3)(w-1)} \\
&= \frac{(2w-2)-(5w+15)}{(w+3)(w-1)} \\
&= \frac{-3w-17}{(w+3)(w-1)} = -\left(\frac{3w+17}{(w+3)(w-1)}\right)
\end{aligned}$$

Click on the green square to return □

Exercise 4(a) Taking common denominators:

$$\begin{aligned}
\frac{a}{x-2}+\frac{b}{x+2} &= \frac{4x}{(x-2)(x+2)} \\
\frac{a(x+2)}{(x-2)(x+2)}+\frac{b(x-2)}{(x-2)(x+2)} &= \frac{4x}{(x-2)(x+2)} \\
\frac{(a+b)x+(2a-2b)}{(x-2)(x+2)} &= \frac{4x}{(x-2)(x+2)}
\end{aligned}$$

so that $(a+b)x+(2a-2b)=4x$. Equating coefficients in this case gives

$$\begin{aligned}
a+b &= 4 \qquad \text{(coefficients of } x\text{)} \\
2a-2b &= 0 \qquad \text{(constant terms)}
\end{aligned}$$

Solving this set of equations gives $a=2$, $b=2$. Hence

$$\frac{2}{x-2}+\frac{2}{x+2}=\frac{4x}{(x-2)(x+2)}$$

Click on the green square to return □

Exercise 4(b) Taking common denominators:

$$\begin{aligned}
\frac{a}{z-3}+\frac{b}{z+2} &= \frac{z+7}{(z-3)(z+2)}\\
\frac{a(z+2)}{(z-3)(z+2)}+\frac{b(z-3)}{(z-3)(z+2)} &= \frac{z+7}{(z-3)(z+2)}\\
\frac{(a+b)z+(2a-3b)}{(z-3)(z+2)} &= \frac{z+7}{(z-3)(z+2)}
\end{aligned}$$

so that $(a+b)z+(2a-3b)=z+7$. Equating coefficients in this case gives

$$\begin{aligned}
a+b &= 1 \qquad \text{(coefficients of } z)\\
2a-3b &= 7 \qquad \text{(constant terms)}
\end{aligned}$$

Solving this set of equations gives $a=2$, $b=-1$. Hence

$$\frac{2}{z-3}-\frac{1}{z+2}=\frac{z+7}{(z-3)(z+2)}$$

Click on the green square to return □

Exercise 4(c) Taking common denominators:

$$\begin{aligned}
\frac{a}{w-4}+\frac{b}{w+1} &= \frac{3w-2}{(w-4)(w+1)}\\
\frac{a(w+1)}{(w-4)(w+1)}+\frac{b(w-4)}{(w-4)(w+1)} &= \frac{3w-2}{(w-4)(w+1)}\\
\frac{(a+b)w+(a-4b)}{(z-3)(z+2)} &= \frac{3w-2}{(w-4)(w+1)}
\end{aligned}$$

so that $(a+b)w+(a-4b)=3w-2$. Equating coefficients in this case gives

$$\begin{aligned}
a+b &= 3 \qquad \text{(coefficients of } w\text{)}\\
a-4b &= -2 \qquad \text{(constant terms)}
\end{aligned}$$

Solving this set of equations gives $a=2$, $b=1$. Hence

$$\frac{2}{w-4}+\frac{1}{w+1}=\frac{3w-2}{(w-4)(w+1)}$$

Click on the green square to return □

Solutions to Quizzes

Solution to Quiz: The numerator and denominator respectively factorise as

$$t^2 + 3t - 4 = (t-1)(t+4) \quad \text{and} \quad t^2 - 3t + 2 = (t-1)(t-2)$$

so that

$$\begin{aligned} \frac{t^2 + 3t - 4}{t^2 - 3t + 2} &= \frac{(t-1)(t+4)}{(t-1)(t-2)} \\ &= \frac{t+4}{t-2} \end{aligned}$$

where the factor $(t-1)$ has been cancelled from the first equation.

End Quiz

Solution to Quiz:

For $\dfrac{x - \frac{1}{x+1}}{x-1}$, the common multiplier is $(x+1)$. Multiplying the numerator and the denominator by this gives:

$$\begin{aligned}
\frac{x - \frac{1}{x+1}}{x-1} &= \frac{(x+1)\left(x - \frac{1}{x+1}\right)}{(x+1)(x-1)} \\
&= \frac{(x+1)x - (x+1)\left(\frac{1}{(x+1)}\right)}{(x^2-1)} \\
&= \frac{x^2+x-1}{x^2-1}
\end{aligned}$$

End Quiz

Solution to Quiz: Writing all the fractions with a common denominator

$$\begin{aligned}
\frac{4x+5}{(2x+1)(x+2)} &= \frac{\mathbf{a}}{2x+1} + \frac{1}{x+2} \\
&= \frac{\mathbf{a}(x+2)}{(2x+1)(x+2)} + \frac{(2x+1)}{(2x+1)(x+2)} \\
&= \frac{\mathbf{a}x + 2\mathbf{a} + 2x + 1}{(2x+1)(x+2)} \\
&= \frac{(\mathbf{a}+2)x + (2\mathbf{a}+1)}{(2x+1)(x+2)}
\end{aligned}$$

so that $(\mathbf{a}+2)x + (2\mathbf{a}+1) = 4x + 5$. This gives two equations

$$\begin{aligned}
\mathbf{a} + 2 &= 4 \qquad \text{coefficients of } x \\
2\mathbf{a} + 1 &= 5 \qquad \text{constant coefficients}
\end{aligned}$$

The solution is $\mathbf{a} = 2$. End Quiz

Solution to Quiz: Writing all the fractions with a common denominator

$$\begin{aligned}
\frac{a}{2x-3}+\frac{b}{3x+4} &= \frac{x+7}{(2x-3)(3x+4)} \\
\frac{a(3x+4)}{(2x-3)(3x+4)}+\frac{b(2x-3)}{(2x-3)(3x+4)} &= \frac{x+7}{(2x-3)(3x+4)} \\
\frac{(3a+2b)x+(4a-3b)}{(2x-3)(3x+4)} &= \frac{x+7}{(2x-3)(3x+4)}
\end{aligned}$$

so that $(3a+2b)x+(4a-3b)=x+7$. This gives two equations

$$\begin{aligned}
3a+2b &= 1 \qquad \text{(coefficients of } x\text{)} \\
4a-3b &= 7 \qquad \text{(constant terms)}
\end{aligned}$$

Solving these gives $a=1$, $b=-1$. End Quiz