Recognition of Arrows in Line Drawings based on the Aggregation of Geometric Criteria using the Choquet Integral

Laurent Wendling and Salvatore Tabbone
LORIA,
Campus scientifique, BP 239
54 506 Vandœuvre-les-Nancy Cedex France
{wendling,tabbone}@loria.fr

Abstract

A new way to detect arrows in line drawings is proposed in this paper. Our approach is based on the definition of the structure of such a symbol. Signatures of angular areas are computed and axiomatic properties and geometric characteristics are checked using the Choquet integral. Finally an experimental application on line-drawing documents shows the interest of our approach.

1. Introduction

Despite the large amount of technical documentation floating around, there have been relatively few accurate studies only focused on the detection of arrows in line drawings. It is well known that such a symbol brings precious information for the understanding of the document under consideration (objects pointed out, associated text boxes...) and in many recognition systems, it is important to have an accurate and powerful operator dedicated to the retrieval of this particular symbol. Dori and Velkovitch [6] have proposed a method for the location of dimensioning text from engineering drawings based on arrowhead recognition [5]. As in [14] the results provided are interesting but application dependent and required arrows having low distortions and belonging to a standard type as ISO or ANSI. In structural pattern recognition field the methods are usually based on graph matching to identify handwriting symbols in graphic documents [1, 11, 17]. In these approaches some rules are defined from structural primitives of features with a cost function associated to. Nevertheless the methods used are generally sensitive to the noise and the deformation of the objects. Valveny and Marti have recently proposed a deformable template Matching [18]. Such an approach is based on a probabilistic model composed of lines. Nonetheless this method is dedicated to symbols described by a set of segments and it is not easy to extend it to manage with binary objects. Moreover the application of graph matching approaches on large documents should required several computing to take into account occluded objects (subgraph matching). In this paper a generic approach dedicated to the definition and the recognition of arrows is presented.

2. Arrow Properties

In this paper an arrow is assumed to be the union of two geometric parts: an isocel triangle T – defined by three points $A(x_a, y_a)$, $B(x_b, y_b)$ and $C(x_c, y_c)$ – linked to a rectangle $R = (EFGH)$ – defined by four points $E(x_e, y_e)$, $H(x_h, y_h)$, $H(x_f, y_f)$ and $G(x_g, y_g)$ –. We set $c = d(A,B) = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$, $b = d(B,C)$ and $a = d(A,C)$ (figure 1).

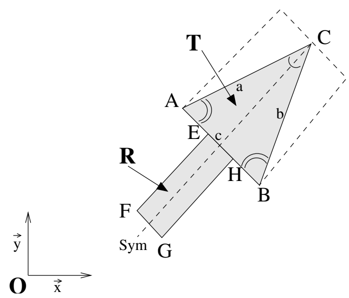

Figure 1. Arrow Description.

By definition a triangle is a 3-sided polygon having some basic geometric properties like [19]:
[P1] All triangles are convex.
[P2] A triangle T with two sides equal is called isocel.
[P3] The sum of the angles of a triangle is π.
[P4] 3 angles specify a triangle only modulo a scale size.
Beyer [3] and Baker [2] have given 110 formula to defined

Proceedings of the Seventh International Conference on Document Analysis and Recognition (ICDAR 2003)
0-7695-1960-1/03 $17.00 © 2003 **IEEE**

the area of a triangle. In this paper the Heron's formula is used to compute this feature:

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

with $s = \frac{1}{2}(a+b+c)$ the semi-perimeter.
The description of an arrow requires also the following property (symmetry):
[P5] An isocel triangle (not equilateral) has a unique angle bisector Sym, passing by C, which split it into two symmetrical parts. This bisector is also the median of the rectangle in a right built arrow.

3. Angular Signature Description

An arrow is assumed to be an isocel triangle T with a linked rectangle R, verifying [P5]. Let us take the point A of T. $Sec(a,c)$ is assumed to be the sector defined from $[A,C]$ and $[A,B]$ which includes T from A. We set $\theta_B^A = \arctan(y_B - y_A/x_B - x_A) + k\pi$ with $x_B - x_A \neq 0$, the angle between (A,B) and (O,x) and $\theta_C^A = \arctan(y_C - y_A/x_C - x_A) + k\pi$ with $y_C - y_A \neq 0$ the angle between (A,C) and (O,x).
Remark: in the following expressions, the value of angle θ is always supposed included in $Sec(a,c)$ to simplify the expressions $-$ that is extended in this paper to any sector $-$.
From [P1] we have for any segment joining two points in T, every point on the segment must also lie within T. That is extended to:
[P6] Let us take the pencil of lines, noted $\mathcal{L}^A = \{D_\theta^A\}_{\theta\in[0,\pi]}$, including A. There exists a set of segments I_A^θ beared by pencil of lines contained in $Sec(a,c)$ describing enterly T .
Let us note C_A the circle centered in A of radius $r = \max(a,c)$ (figure 2). All the segments I_A^θ, verifying [P6], are including in C_A: $V_A = \{I_A^\theta\}_{\theta\in[\theta_B^A,\theta_C^A]}$ is set as the definition of T from A (figure 2).

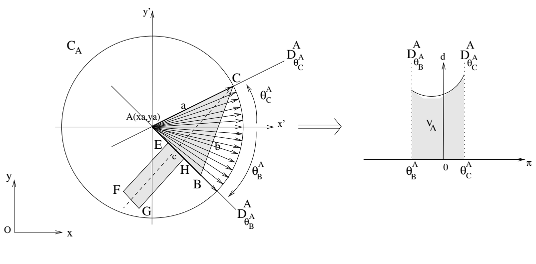

Figure 2. Signature V_A.

The same scheme is performed to define V_B. V_C includes also the definition of R (that is using C_C centered in C of radius $r = \max(a, d(C,F))$). In this case T and R are enterly described ($(CEFGH)$ is convex).

In image processing V_A, V_B and V_C will be computed in raster data and matched with their theorical approximation function. The definition of such functions is now presented. Let us consider any triangle T' defined by three points: X_1, X_2 and X_3. X_1 is assumed to be the origin of the orthogonal frame, θ' and θ'' the angles described by $[X_1,X_2]$ and $[X_1,X_3]$ from this frame. We also set $x = d(X1,X2)$ and $y = d(X1,X3)$ with d the Euclidean distance. The aim is to reach a continuous approximation, noted S_X, of the signature V_X previously described. That is to have a new representation of the segment $[X2,X3]$ from $X1$.
The continuous function $S_{X_1}(\theta) : [\theta',\theta''] \to \mathbb{R}_*^+$, associated to V_{X_1} is given by:

$$f(\theta, <x,y,\theta',\theta''>) = \frac{x\cdot y\cdot \sin(\theta'+\theta'')}{x\cdot\sin(\theta-\theta') - y\cdot\sin(\theta-\theta'')}$$

Let us now consider the points A, B and C of triangle T.
The continuous function $S_A(\theta) : [\theta_B^A,\theta_C^A] \to \mathbb{R}_*^+$, associated to V_A, is:

$$S_A(\theta) = f(\theta, <a,c,\theta_B^A,\theta_C^A>)$$

The continuous function $S_B(\theta) : [\theta_C^B,\theta_A^B] \to \mathbb{R}_*^+$, associated to V_B, – in this case, the orthogonal frame is referred to $B(x_B,y_B)$ –, corresponds to:

$$S_B(\theta) = f(\theta, <b,c,\theta_C^B,\theta_A^B>)$$

The continuous function $S_C(\theta) : [\theta_A^C,\theta_B^C] \to \mathbb{R}_*^+$ is defined as follows $(a=b)$:
If R is an empty rectangle:

$$S_C(\theta) = f(\theta, <a,a,\theta_A^C,\theta_B^C>)$$

with $\theta_A^C \leq \theta \leq \theta_B^C$ is defined from the frame $C(x_C,y_C)$.
else ($R \neq \emptyset$), 5 triangles are processed (see figure 3):
$a' = d(C,E) = d(C,H)$, $a'' = d(C,F) = d(C,G)$
The expression of $S_C(\theta)$ is given by:

$$S_C(\theta) \to \begin{cases} \theta_A^C \leq \theta < \theta_E^C & f(\theta, <a,a',\theta_A^C,\theta_E^C>) \\ \theta_E^C < \theta \leq \theta_F^C & f(\theta, <a',a'',\theta_E^C,\theta_F^C>) \\ \theta_F^C < \theta \leq \theta_G^C & f(\theta, <a'',a'',\theta_F^C,\theta_G^C>) \\ \theta_G^C < \theta \leq \theta_H^C & f(\theta, <a'',a',\theta_G^C,\theta_H^C>) \\ \theta_H^C < \theta \leq \theta_B^C & f(\theta, <a',a,\theta_H^C,\theta_B^C>) \end{cases}$$

[P7] $V_A \Leftrightarrow S_A$, $V_B \Leftrightarrow S_B$ and $V_C \Leftrightarrow S_C$
□ Proofs of $\Rightarrow$ and $\Leftarrow$ are directly based on the differential definition of V_A (V_B and V_C) and S_A (S_B and S_C).

Now we can consider the global signature S associated to an arrow as the union of signatures: Let t_π be a translation of π, $S(T^{ABC}) = S_A \cup t_\pi(S_C) \cup S_B$ is a complete signature of T. S is unique and we can directly deduce from [P7] that: Let t_π be a translation of π, $V(T^{ABC}) = V_A \cup t_\pi(V_C) \cup V_B$ is a complete signature of T ($V \Leftrightarrow T$).

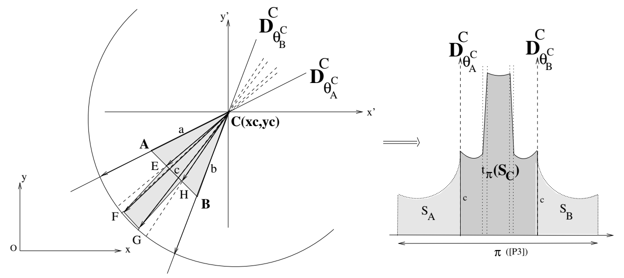

Figure 3. Signature $S_C(\theta)$.

The following properties are interesting to estimate criteria:

[P8] $V_A \cap t_\pi(V_C) = V_B \cap t_\pi(V_C) = \{c\}$, $V_A \cap V_B \cap V_C = \emptyset$.

[P9] $|V_A| = |V_B|$.

□ From [P2,P6,P7], $\int_{\theta_C^A}^{\theta_B^A} S_A(\theta)d\theta = \int_{\theta_C^B}^{\theta_A^B} S_B(\theta)d\theta$ is true.

[P10] There exists a symmetry, noted Sym, such that $Sym(S_A) = S_B$.

□ Let us consider V_A and V_B as being two sub-vector fields. From [P2,P5] we have $\forall I_A^\theta \in V_A, \exists$ a unique $I_B^{\theta'} \in V_B$ such that $Sym(I_A^\theta) = I_B^{\theta'}$ and $Sym(I_B^{\theta'}) = I_A^\theta$.

[P11] S_C is a symmetric map. That is $\forall I_C^\theta \in V_C, \exists$ a unique $I_C^{\theta'} \in V_C$ such that $Sym(I_C^\theta) = I_C^{\theta'}$. There also exists a unique I_C^θ such that $Sym(I_C^\theta) = I_C^\theta$.

□ From [P2,P5,P7]; let us consider $R = \emptyset$, it is easy to show that $S_C(\theta^C - \theta'_C) = S_C(\theta^C + \theta'_C)$ with $\theta'_C = \frac{\theta_A^C + \theta_B^C}{2}$. If $R \neq \emptyset$ the reasoning is similar for each symmetric pair.

The signatures V and S keep fundamental geometric properties, useful in a pattern recognition process, as:

[P12] S is invariant by translation.

□ Let $t_{\vec{u}}$ be a translation of vector $\vec{u}$:
$f(\theta, < d(X1,X2), d(X1,X3), \theta', \theta'' >) =$
$f(\theta, < d(t_{\vec{u}}(X1), t_{\vec{u}}(X2)), d(t_{\vec{u}}(X1), t_{\vec{u}}(X3)), \theta', \theta'' >)$

[P13] The angle of rotation can be deduced in S.

□ A rotation of α is assumed to be a translation of α in S:
$f(\theta, < x, y, \theta' + \alpha, \theta'' + \alpha >) = f(\theta, < x, y, \theta', \theta'' >)$
implies f is invariant by rotation.

[P14] S-normalized is invariant by scale factor.

□ Let k be the homothecy factor, we have:
$f(\theta, < k \cdot x, k \cdot y, \theta', \theta'' >) = k \cdot f(\theta, < x, y, \theta', \theta'' >)$

4. Arrow Detection

4.1. Criteria

Let A, B, and C be a triplet of points to valuate. The previous part allows to define several measures to check an arrow. The set of criteria used here is the following:
$N = \{Sym, Card_{AB}, \bar{O}ver, Area, Proto\}$

Sym	$\max_{t\in[1,p/2]}\left\{\frac{\sum_{i=1,p}\min(V^{t+i},V^{t-i})}{\sum_{i=1,p}\max(V^{t+i},V^{t-i})}\right\}$
$Card$	$\min(\lvert V_A\rvert,\lvert V_B\rvert)/\max(\lvert V_A\rvert,\lvert V_B\rvert)$
$\bar{O}ver$	$1.-\frac{\sum_{i=1,p}\inf\{V_A^i,V_B^i,t_\pi(V_C^i)\}-2\cdot d(A,B)}{\sum_{i=1,p}\sup\{V_A^i,V_B^i,t_\pi(V_C^i)\}}$
$Area$	$1.-(K-H)/H$
$Proto$	$D(V,\bar{S})$

The degree of symmetry (Sym) is computed to have an approximation of [P10] and [P11]. From [P9] the cardinality of V_A is close to the one of V_B ($Card_{AB}$). An estimation of the degree of non-overlap ($\bar{O}ver$) is required to check [P8]. Let K be the common area carried out from the scans during the definition of V_A, V_B and V_C, it should be close to the calculation of the Heron formula H from these points ($Area$). At last, the numerical signature V given by the triplet is matched with its theorical prototype S ([P7]). D is a distance and $\bar{S}$ is a discrete approximation of S.

4.2. Choquet Integral

A first idea to merge these criteria may be to use a weighted sum. We can directly think that Sym, $\bar{O}ver$ and $Proto$ are the main criteria but what is the most important? Furthermore the criteria should be in correlation as they are defined from the basic structure of the arrow. Such a dependence can not be formalized using classical combinaisons. Our aggregation was based on the Choquet integral concept whose use was proposed by many authors (see [7]).

Let $N = \{1, 2, \ldots, n\}$ be a finite set of n criteria $-$ $n = 5$ in our study $-$ and let $A = \{a, b, ..\}$ be a finite set of alternatives $-$ that is the values reached from all the triplets of points found in a document $-$.

Let v be a monotonic set function $-$ also called fuzzy measure $-$ on N such that [12, 16]: $v(\emptyset) = 0$, $v(N) = 1$ and $S \subseteq T \Rightarrow v(S) \leq v(T)$. Let us now consider an alternative $x^i = (x_1^i, \ldots, x_n^i)$, where for any criterion j, x_j^i is the score of i related to j [10].

$\forall v \in \mathcal{P}_N$, the Choquet integral of $x \in \mathbb{R}^n$ is defined by:
$\mathcal{C}_v(x) := \sum_{i=1}^n x(i)[v(A_{(i)}) - v(A_{(i+1)})]$
where $(.)$ is a permutation of N, carried out from the values obtained by the application of each criterion on a triplet, such that $x(1) \leq \ldots \leq x(n)$ and $A(i) = \{(i), \ldots, (n)\}$ represents the $[i..n]$ associated criteria in increasing order.

By definition $v(\emptyset) = 0$ and $v(\{1, \ldots, n\}) = 1$. The Lemke method is generally used to set the $2^N - 2$ remaining values [7, 12] but can provide incoherent results [8]. The fuzzy measure was defined from samples (arrows and other symbols) from the optimal algorithm proposed by Grabisch [8]. Significant criteria, negative and positive interactions can be established from the $2^N - 2$ values reached. The importance of each criteria [7] is based on the definition proposed by Shapley [15] extended to fuzzy measure in [13]. An importance index greater than 1 describes an attribute more important than the average:

Criterion	Sym	$Card$	$\bar{O}ver$	$Area$	$Proto$
Sh. value	0.72	0.51	**1.03**	0.76	**1.97**

This definition has been extended in [13] to measure the interaction index between criteria (values in $[-1, 1]$):

S, C	-0.05	S, O	-0.45	S, A	-0.14
S, P	**0.55**	$C, \bar{O}$	-0.11	C, A	0.19
C, P	**0.88**	$\bar{O}, A$	**0.54**	$\bar{O}, P$	0.02
A, P	0.41	–	–	–	–

Such tables show that the main criteria are $\bar{O}ver$ and $Proto$ (not Sym!) and high correlations exist between criteria (between $Card$ and $Proto$ and $Area$ and $\bar{O}ver$).

5. Arrow Recognition

5.1. Cardinal lists

For efficiency reasons, we use Bresenham's algorithm [4] in our implementation, as it is a fast method which minimizes error in drawing lines on integer grid points. First, let us suppose arrows are not occluded in the drawing and the angle of the sector defined from C is acute ($Sec(a < b) < \pi/2$). Four scans of the plane from four cardinal directions are enough to locate any edge as a triangle is convex by definition ([P1]). All the segments $V_{I_{ij}}$ ([P6]) are labeled with a level greater to 1 during the definition of the signature to limit the search to the location of new starting points. If the angle is obtuse the method may require the study of double points to rightly locate C.

5.2. Robustness to Noise

The starting area from A, B and C might be moved using noisy data. A disk search area ($r = 1$) is set to compute the better starting points. Figure 4 shows a degraded arrow and the superimposed signatures reached. We can remark that the shape of the two signatures are close. Such a result is underligned by the high values reached from each criterion.

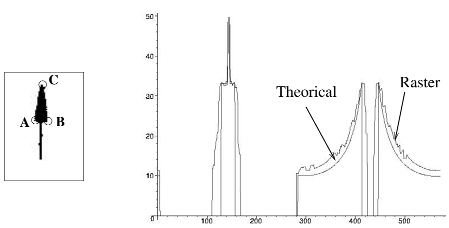

Figure 4. Signatures of Noisy Arrow.

Criterion	Sym	$Card$	$\bar{O}ver$	$Area$	$Proto$
Values	0.88	0.85	0.94	1.	0.88

5.3. Occlusions

If A, B and C are not occluded, our approach allows to find correctly an arrow even if parts of the symbol are occluded (figure 5.a) because the additive data do not interfer on the calculation of the general structure. However the edges of arrows can be occluded in line drawings (figure 5.b). Two approaches were studied to estimate the location of occlusion points. In the first hand, the new points were directly calculated from the intersection of the lines defined from the double points found in the cardinal lists (figure 5.c). In the other hand skeletonization and pruning steps [9] are required to manage with more important occlusions. In these case the junction edges found (figure 5.d) are assumed to be potential arrow points. The list of occlusion points found is appended to each cardinal list.

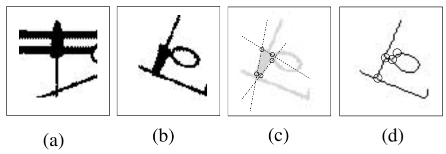

Figure 5. Occulted Arrows.

5.4. Rectangle Estimation

The rectangle (EFGH) is taken into account during the calculation of overlap and symmetry criteria. Nevertheless using noisy data an estimation of the rectangle is required to compute $Proto$ properly. According to us an arrow without rectangle is assumed to be a degraded arrow and should have a lower recognition rate. Despite this consideration the lengther the rectangle, the less the accuracy of $Proto$ estimation is. Two boundaries are defined from V_C where the rectangle should be considered (see figure 6).

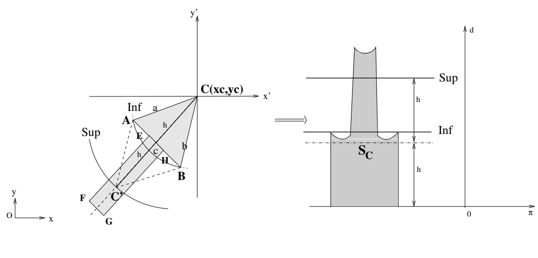

Figure 6. Rectangle signature boundaries.

h is the altitude. The width [EH] of the rectangle is directly computed from V_c as the maximal symmetric pair from Inf up to Sup (calculated from the rhombus $(CBC'A)$). The new V_C signature is truncated of greater values. Inf may be greater that Sup when handling with obtuse angles. In this case, the two values are permuted.

5.5. Triplet Checking

The aim is to define a set of triplets to valuate from four lists of points. So we have set some basic constraints to decrease drastically the number of choices:
• The points should be in the same connected component.
• Each point should be "visible" by the others.
• The selected points belong to distinct lists.
• If an arrow is found the points are removed from the cardinal lists.
When a triplet is found, three combinations of points are evaluated from the Choquet integral to determine which point is the more probable head of the arrow.

5.6. Line Drawing Application

Our approach has been applied on several types of line drawings with success and problems only occur with large occlusions. In the figure 7.a the circles represent the list of triplets found from the cardinal lists, the squares refer to the detected arrows (scores greater than 0.75 − fixed by the learning step −) and the letter f is a symbol badly recognized using classical weighted sum. Another example is provided in the figure 7.b.

a)

b)

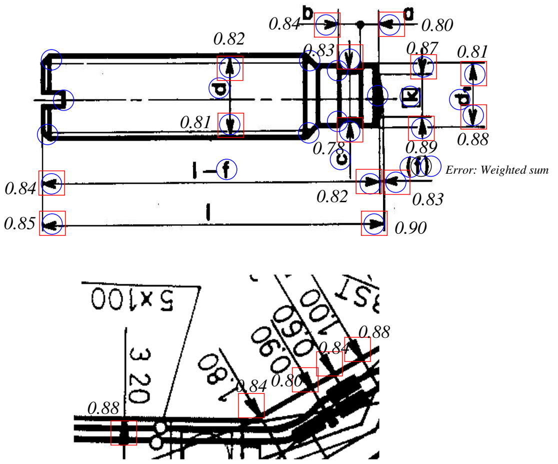

Figure 7. Detection of Arrows.

6. Conclusion

We have shown in this paper that the definition of both angular and theorical signatures and the checking of axiomatic properties allowed to reach a robust detection of arrows. The experimental study shows the promising aspect of our approach. Currently, we focus on sub-pixel definition to decrease the aliasing given by bresenham's algorithm and we want to extend our approach to the extraction of text boxes in line drawings.

References

[1] Ah-soon C. and Tombre K., Architectural symbol recongition using a network of constraints., *Pattern Recognition Letters*, 22(2):231–248, 2001.

[2] Baker M., A Collection of Formulae for the Area of a Plane Triangle, *Ann. Math*, 1:134–138, 1884.

[3] Beyer W.H., CRC Standard Mathematical Tables, *28th ed. Boca FL: CRC Press*, 123–124,1987.

[4] J. E. Bresenham, Algorithm for Computer Control of a Digital Plotter, *IBM Systems Journal*, 4(1):25–30, 1965.

[5] I. Chai and D. Dori, Extraction of text boxes from engineering drawings, *Proc. SPIE 1661*, 38-49, 1992.

[6] Dori D. and Y. Velkovitch, Segmentation and Recognition of Dimensioning Text from Engineering Drawings, *CVIU*, 69(2):196–201, 1998.

[7] M. Grabisch, The application of fuzzy integral in multicriteria decision making, European journal of operational research, vol. 89:445–456, 1995.

[8] M. Grabisch, A New Algoirthm for Identifying Fuzzy Measures and Its Application to Pattern Recognition, FUZZ'IEEE 95, vol. 1:145–150, 1995.

[9] C.P. Lai and R. Kasturi Detection of Dimension Sets in Engineering Drawings, *ICDAR'93*, pp: 606-613, 1993.

[10] J.-L. Marichal and M. Roubens, Determination of weights of interacting criteria from a reference set, *European Journal of Operational Research*, 124(3):641–650, 2000.

[11] Messmer B.T. and Bunke H., Automatic learning and Recognition of graphical symbols in engineering drawings, *In kasturi and Tombre editors, Graphics Recognition: Methods and Applications*, Springer-Verlag, 123–134, 1987.

[12] T. Murofushi and M. Sugeno, An interpretation of fuzzy measure as an integral with respect to a fuzzy measure, *Fuzzy Sets ans Systems*, 29:201–227, 1989.

[13] T. Murofushi and Soneda S., Techniques for reading fuzzy measure (iii): interaction index, 9th Fuzzy System Symposium, 693–696, in Japanese, 1993.

[14] G. Priestnall, R.E Martson and D.G. Elliman Arrowhead recognition during automated data capture, *Pattern Recognition Letters*, 17:277–286, 1996.

[15] L.S. Shapley, A value for n-person games, in: H.W. Khun and A.W. Tucker Eds., *Contributions to the Theory of Games, Vo. II, (Annals of Mathematics Studies, 28)*, Princeton University Press, 307–317, 1953.

[16] M. Sugeno, Fuzzy measures and fuzzy integrals – A survey, in: Gupta, Saridis and Gaines (eds.), *Fuzzy Automata and Decision Processes*, 89–102, 1977.

[17] C.C. Tappert, C.Y. Suen and T. Wakahara, The state of the Art in Online Handwriting Recognition, *In IEEE Transactions of PAMI*, 12:787–808, 1990.

[18] E. Valveny, E. Marti, Learning of Structural Descriptions of Graphic Stymbols using Deformable Template Matching, *In* 6^{th} *ICDAR*, 455–459, 2001.

[19] Eric W. Weissten, CRC Concise Encyclopedia of Mathematics, Chapman & Hall/CRC, 1834–1838, 1999.

Proceedings of the Seventh International Conference on Document Analysis and Recognition (ICDAR 2003)
0-7695-1960-1/03 $17.00 © 2003 **IEEE**